Financial Modelling Insights

The inflation index

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1 Introduction

Here we elucidate the process required to produce financial scenarios of the 'inflation index' variable. Such a simulation has been performed within the risk neutral framework.

Inflation plays a key role in financial markets, and indeed this variable is one of the targets of the monetary policies of central banks. Moreover, due to the impact of inflation on investors' behaviour, in recent years, an increasing number of instruments have been quoted on financial markets. Such instruments allow to mitigate risks and to seize opportunities linked to inflation.

Inflation has a number of unique features: its value is set by measures of real economy whereas financial markets indicate expectations on its future values. These and other features generate some constraints on inflation models, thus particular attention is required when implementing a new one. For instance, a coherence constraint between inflation levels and interest rate variables is crucial for many inflation models.

In recent years, some economies have witnessed deflation trends: due its peculiarity, this dynamic has sparked new interest in modelling inflation.

In more practical terms, we will argue the steps needed to simulate inflation using the Jarrow-Yildirim model.

2 The inflation index

The inflation rate is defined as the percentage change of a reference index[1], namely the Consumer Price Index (CPI), which is based on a basket of goods and services. In Italy the index is estimated by Istat[2], whereas the European index is published by Eurostat[3] a few weeks after the end of the month. Due to the increasing number of inflation-linked instruments, market

[1] Quoted inflation rate is computed by taking as reference level the inflation index published two months in advance.
[2] Italian National Institute of Statistics.
[3] Statistical office of the European Union.

players have been displaying an increasing interest in such a variable.

Inflation-linked bonds have been issued since the 1980s, whereas the market has become liquid since the 2000s, particularly in Europe. The market of inflation options has shown an upward evolution in recent years, but liquidity in this market is still limited and often a few 'strike prices' are traded (ECB, 2013). A comprehensive guide to this market can be found in Deacon, Derry and Mirfendereski (Deacon, Derry, & Mirfendereski, 2004).

The inflation rate shows a behaviour influenced by interest rates (Fama, 1975), i.e. the levels of interest rate on the swap market.

Indeed, investors monitor inflation rate levels in order to keep their purchasing power under control. Hence, the 'cost of money', which investors focus on, is inflation-adjusted. Such a variable, referred to as 'real rate', is obtained as the difference between nominal rates and inflation rates.

Thus, the risk associated with real interest rates depends on both dynamics: nominal interest rates and inflation rates. These two variables show a correlated behaviour due to their close relationship in macroeconomic dynamics (Lee, 1992), therefore, interest rates should also be taken into account in order to simulate inflation scenarios. This feature is implemented by the inflation model discussed in the following section: the Jarrow-Yildirim model.

In literature, inflation is widely analysed in macroeconomic and pricing terms alike. For instance, there are macroeconomic works concerning the relation that holds between inflation and growth (Khan & Ssnhadji, 2001) and relating to the estimation of market inflation expectations by using inflation swap rates (Hurd & Relleen, 2006).

With respect to the pricing of inflation instruments, different models and approaches have been developed: models based on the analogy with foreign exchange dynamics (Hughston, 1998), models that implement a discrete-time stochastic approach (Macrina & Hughston, 2008), models exploiting the no-arbitrage relationship between zero coupon and the 'year on year' swaps (Belgrade, Benhamou, & Koehler, 2004).

2.1 The model: Jarrow-Yildirim

Jarrow and Yildirim (Jarrow & Yildirim, 2003) proposed a model based on a foreign-currency analogy that has become the benchmark approach for pricing purposes, thanks to its mathematical tractability.

The Jarrow-Yildirim (JY) model delineates inflation dynamics via a process that involves the evolution of three correlated variables: nominal interest rate *n(t)*, real interest rate *r(t)* and inflation index *I(t)*. Interest rates are described by processes set out by Hull and White (J.Hull & White, 1990), whereas the inflation index is a Geometric Brownian Motion, whereby the drift is the difference between the nominal interest rate and the real one under the risk neutral measure. The outputs of the model are: the inflation index, the spot forward nominal and real rate.

A cardinal feature that makes this model easy to use, is the presence of an analytical formula able to reproduce the distribution of inflation at different time horizons (Brigo & Mercurio, 2007). This feature is quintessential in model calibration on market data.

Viewed through the lens of the practitioner, one drawback of this model lies in the direct description of real interest rates; this variable is not quoted on financial markets, but is inferred from other information, therefore its volatility does not constitute a clear basis of comparison.

2.2 The calibration process

Within a risk-neutral framework model, parameters must reflect the information available on the market, specifically the views on future values and volatilities implied in inflation-linked instruments at different maturities. In literature, the process of parameter estimation is referred to as calibration and represents a critical operative step for generating reliable financial scenarios.

The aim of the calibration process is to obtain a set of parameters which minimise the differences between the prices observed on the market and those predicted by the model. This purpose may be attained by selecting liquid market prices of financial inflation-linked instruments at a specific reference date.

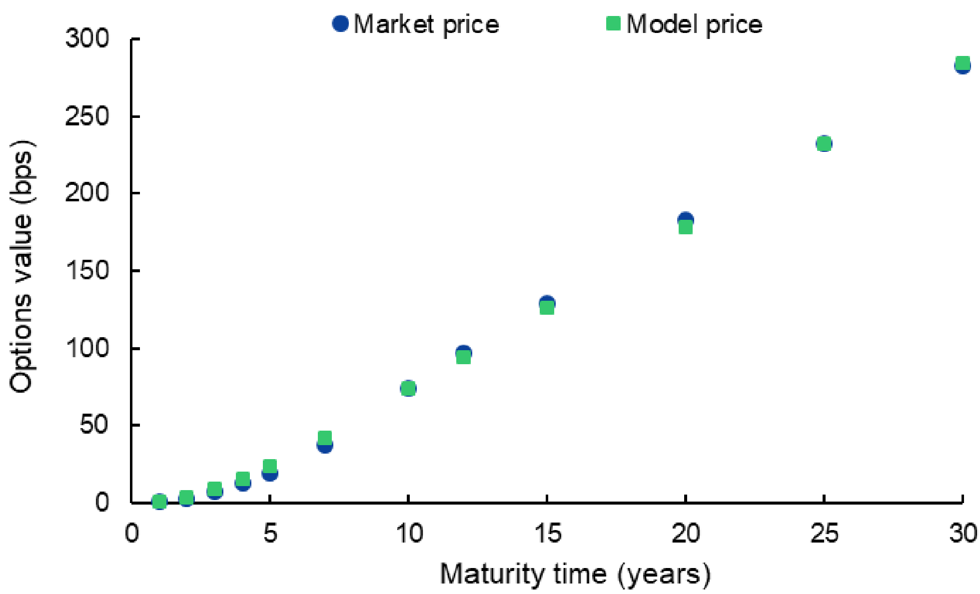
The calibration process results in an optimisation procedure, thus the method and the technical choices adopted for the optimisation task may produce relevant effects on the fitting quality.

The selection of a specific set of instruments for model calibration is determined by: the particular simulation purpose (i.e. pricing of portfolio with some specific characteristics) and the availability of market data. For instance, the expected trend of the inflation index is derived from inflation-linked bonds or zero-coupon inflation swaps, whereas inflation caps and floors traded for several maturities are used to calibrate the volatility of the model.

Thus, the reliability of simulated scenarios depends on the reliability of market data employed in calibration.

Figure 1 shows the calibration of the JY model on at-the-money floor options. Specifically, a set of options with fixed tenor, maturity increasing from one to 30 years, is represented.

Figure 1
Model calibration based on inflation options: floors with tenor equal to one year

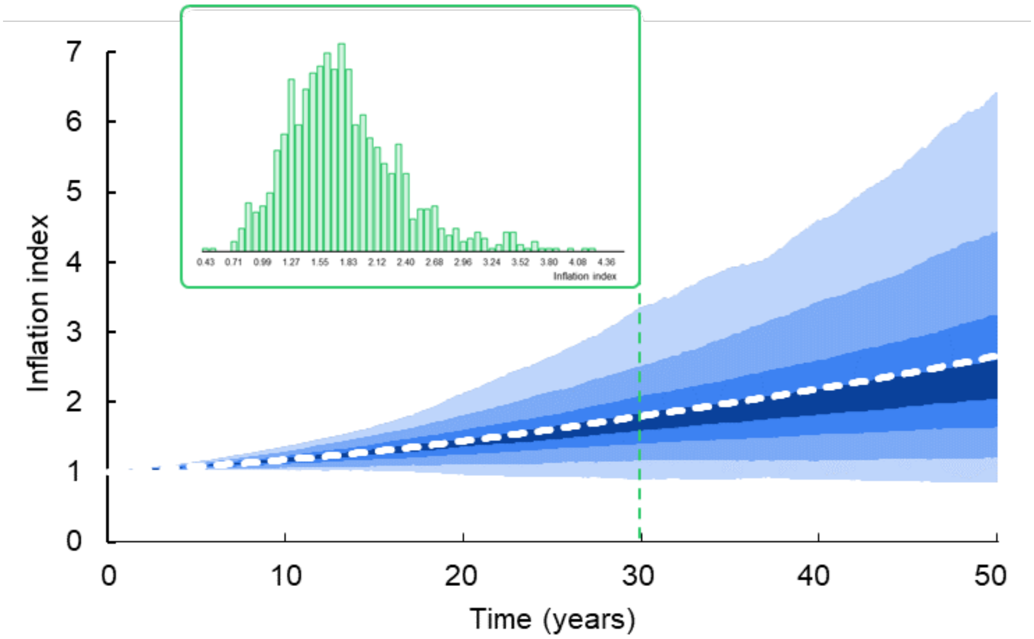


2.3 Testing simulation quality

Once the model has been calibrated on market data, it can be used to generate market consistent scenarios. We refer to scenarios as the output of the simulation method referred to in literature as Monte Carlo (Glasserman, 2003).

Figure 2 shows the simulated inflation index. The time horizon of the simulation is 50 years with 1,000 simulated scenarios. That is to say, the shape of the index distribution is displayed over a 30-year span. This shape is somewhat asymmetric due to the log-normal dynamics of the model.

Figure 2
Scenarios generated for the inflation index (1,000 scenarios)



Following simulation, prior to using the scenarios, the reliability of the latter needs to be validated via various tests. Some of these are outlined below.

First of all, the reliability of the simulation procedure may require verification. In this context, it is possible to verify that the statistical moments estimated from the simulated sample are equal to the analytical ones computed from the model. In order to validate this test, the statistical error associated with the number of scenarios should be considered.

As a next step, the risk neutrality nature of the scenarios may require verification: the evolution of the variable should be coherent with the expected values quoted on the market.

In particular, inflation market rates quoted on the market with different maturities must be reproduced (within the statistical error) by the average inflation rate scenarios resulting from the simulation. Figure 3 shows this verification, whereby market data are always between the confidence bounds at different simulation times: if this requirement is met, the test outcome is positive.

This test is called “Martingale test” and it is a mandatory requirement for the Solvency II Directive.

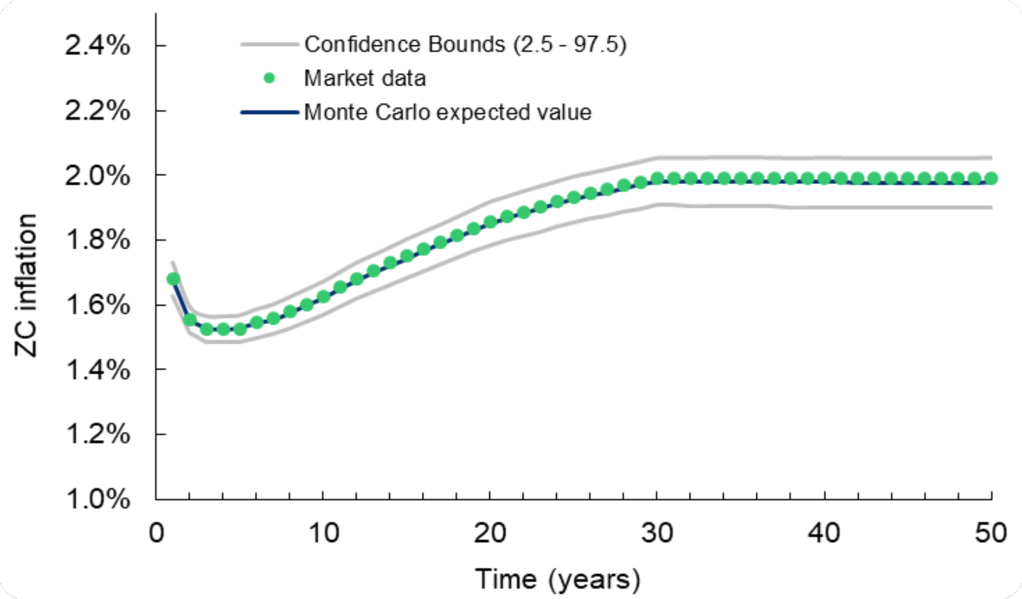


Figure 3
Verification of the risk neutrality nature of generated scenarios

Further investigations conducted on scenarios are also required in order to test their reliability and soundness. For instance, the prices of the instruments obtained via analytical formulas should be consistent with the ones obtained via the Monte Carlo method. This test allows to verify the coherence between the calibration and simulation processes. The test is valid when the price obtained via the Monte Carlo method is reproduced (within the statistical error) by the one obtained via the analytical formula of the model.

These tests ensure the consistency of the generated scenarios with respect to their risk neutrality and the possibility of reproducing market information.

However, some decisions underlying the operational steps (relating to model, data selection for calibration, number of scenarios, simulation steps and time horizon) depend on the specific application of any given scenario: that is to say, the pricing of a specific portfolio. The requirement whereby a unique set of scenarios must be used to price several different instruments often leads to some trade-off between accuracy and time performance. In such cases, the user’s expert assessment of the process becomes an additional factor to be considered for a correct simulation.

3 Conclusions

During the entire simulation procedure, there are many elements to be given consideration in order to generate reliable inflation scenarios in a risk-neutral framework.

Lastly, it is worth emphasising that the procedure may generate scenarios that fail our validation test. Failure of scenario simulation may depend on: model calibration errors, errors relative to data selection for model calibration, errors in implementing the model or in the simulation methodology.

As mentioned, in order to reduce these sources of error procedures for ensuring the reliability of each operational step need to be developed. However, the simulation of reliable inflation scenarios also needs to consider the specific application of the simulated variable, given the wide variety of inflation-linked instruments on the financial markets.

Furthermore, the development of new benchmark models should be considered to describe inflation. Indeed, in coming years, an increasing interest is expected in models and procedures for inflation simulation, owing to the increasing market size of inflation-linked instruments as well as to the increasing interest of central banks in this variable as a target of their monetary policies.

Inflation scenarios can be used in various ways: from a risk neutral point of view, they can be useful for pricing purposes or for simulating inflation linked assets, in particular, for the estimation of capital requirement under Solvency II.

A similar simulation setup can be used for estimating ‘real world scenarios’, namely scenarios consistent with a historical or an expert view on future trends and volatility, which can be relied on to support asset allocation or the estimation of economic capital.

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